THE COUNCIL OF COMMUNITY COLLEGES OF JAMAICA

BACHELOR OF SCIENCE EXAMINATION

SEMESTER I – 2012 DECEMBER

PROGRAMMES:	**MANAGEMENT INFORMATION SYSTEMS**
COURSE NAME: **CODE:**	**CALCULUS I** **(MATH3601)**
YEAR GROUP:	**THREE**
DATE:	**WEDNESDAY, 2012 DECEMBER 12**
TIME:	**9:00 A.M. – 12:00 NOON**
DURATION:	**3 HOURS**
EXAMINATION TYPE:	**FINAL**

This Examination paper has ***4*** *pages*

INSTRUCTIONS:

SECTION B: ANSWER ANY THREE (3) QUESTIONS FROM THIS SECTION.

SECTION B

Instructions: Answer any <u>THREE (3)</u> questions from this section.

<u>Question 1</u>

a. Determine the limit of the following functions:

i. $\lim_{x \to 4} \frac{x^2 - x - 12}{x - 4}$ ***(3 marks)***

ii. $\lim_{x \to \infty} \frac{2x^3 - 5x^2 + 1}{5x^3 - 3x + 7}$ ***(3 marks)***

iii. $\lim_{x \to 2} 7x^3 - 5x^2 + 2x - 4$ ***(2 marks)***

b. Find the vertical and horizontal asymptotes of the following function:

$$f(x) = \frac{3x^2 - 5x + 2}{6x^2 - 5x + 1}$$ ***(5 marks)***

c. i. State the conditions for a function to be continuous at $x = a$ ***(3 marks)***

ii A function f is defined by

$$f(x) \begin{cases} x^2 + 1, & x \le 2 \\ x + 3, & x > 2 \end{cases}$$

Determine if f is continuous at $x = 2$. Give reason for your answer. ***(6 marks)***

iii. Determine the values of x for which the function $f(x)$ is discontinuous.

$$f(x) = \frac{x^2}{x^2 - x - 6}$$ ***(3 marks)***

(Total 25 marks)

Question 2

a. Differentiate $f\left(x\right)=3x^2+4x-7$ from first principle. ***(5 marks)***

b. Differentiate the following with respect to x.

i. $y=\ln\left(7x+6\right)e^{5x-4}$ ***(4 marks)***

ii. $y=\dfrac{x^2+6}{\sqrt{x^2+5}}$, simplify your answer ***(4 marks)***

c. Find the equation of the tangent to the curve $y^3-2y-x^3+x=15$ at the point $\left(2,3\right)$ ***(7 marks)***

d. A curve is defined parametrically by the equations $x=3t^2$, and $y=6t-1$. Find gradient of the curve at the point where $t=2$. ***(5 marks)***

(Total 25 marks)

Question 3

a. Find the following:

i. $\int\sqrt{x}\left(1-x\right)dx$ ***(4 marks)***

ii. $\int e^{\frac{5}{2}x}\,dx$ ***(3 marks)***

iii. $\int\dfrac{4x+6}{x^2+3x+5}$ ***(4 marks)***

b. By using the substitution $u=\sqrt{1-x}$ evaluate $\int_0^1 x\sqrt{1-x}\,dx$ ***(8 marks)***

c. A region is bounded by the curve $y=x^2$, the lines $x=1$, $x=2$ and $y=3x$. Find the area enclosed by this region. ***(6 marks)***

(Total 25 marks)

Question 4

a. Air is being pumped into a spherical balloon at a rate of $8cm^3s^{-1}$. Find the rate of change of the radius of the balloon when the radius of the balloon is 10 cm. Volume of a sphere is $\frac{4}{3}\pi r^3$ ***(6 marks)***

b. Find the equations of the tangent and the normal to the curve $y = \frac{2}{\left(x^2 - 2x - 4\right)^2}$ at the point (3, 2). ***(10 marks)***

c. Determine the constants a and b so that the function $f\left(x\right) = x^3 + ax^2 + bx$ have stationary points when $x = -1$ and $x = 3$. Determine also the nature of these stationary points. ***(9 marks)***

(Total 25 marks)

Question 5

a. Given the $f(x) = 2x^3 - 12x^2 + 18x$

i. Find the coordinates of the stationary points ***(7 marks)***

ii. Determine the nature of each stationary points ***(4 marks)***

iii. Determine where the curve crosses the x and y axis ***(4 marks)***

iv. Sketch the curve of $f(x)$ showing all critical points ***(5 marks)***

b. Evaluate $\int_0^2 \left(6x^2 + x + 5\right)dx$ ***(5 marks)***

(Total 25 marks)

END OF EXAMINATION